2. The hard steels will not withstand a greater number of reversals of the same range of stress than the mild steels if the periodicity of the reversals is great.

"The Equilibrium of Rotating Liquid Cylinders." By J. H. JEANS, B.A., Isaac Newton Student and Fellow of Trinity College, Cambridge. Communicated by Professor G. H. DARWIN, F.R.S. Received March 6,—Read March 20, 1902.

(Abstract.)

The most serious obstacle to progress in the problem of determining the equilibrium configurations of a rotating liquid lies in the difficulty of determining the potential of a mass of homogeneous matter of which the boundary is given. If this boundary is

$$f(x, y, z) = 0 \quad \ldots\ldots\ldots\ldots \quad \text{(i)},$$

the potential will be a unique-valued function of x, y, and z, of which the form will depend solely upon the form of $f(x, y, z)$. This potential must therefore be deducible by some algebraical transformation of the function f.

In the method usually followed the solution is found as a volume integral, the integration extending throughout the surface (i). There is, however, a second method of obtaining this potential, namely, by regarding the potential-function as the solution of a differential equation, subject to certain boundary conditions. This leads directly to a series of algebraical processes, enabling us (theoretically) to deduce the potential by transformation of the function f.

In three-dimensional problems this method is quite impracticable, since it depends upon a continued application of the formula which expresses the products or powers of spherical harmonics as the sum of a series of harmonics.

As soon, however, as we pass to the consideration of two-dimensional problems, the spherical harmonics may be replaced by circular functions of a single variable. The transformation now becomes manageable, and for this reason the present paper deals only with two-dimensional problems, for which a method is developed enabling us to write down the potential by transformation of the equation of the boundary. The method is not of universal applicability, but is adequate to the problem in hand.

The method as applied to the determination of equilibrium configurations is as follows. Starting from the general equation (in polar co-ordinates)

$$r^2 = a_0 + 2a_1 r \cos\theta + 2a_2 r^2 \cos 2\theta + \ldots \ldots\ldots \quad \text{(ii)},$$

we transform by the substitution

$$\xi = re^{i\theta}, \qquad \eta = re^{-i\theta},$$

and attempt to solve the resulting equation explicitly for ξ in the form

$$\xi = b_1 + b_2\eta + b_3\eta^2 + \ldots + \frac{c_1}{\eta} + \frac{c_2}{\eta^2} + \frac{c_3}{\eta^3} + \ldots \ldots \ldots \text{(iii)}$$

this solution being such that the right hand gives the true value of ξ at every point of the surface given by equation (ii). The condition that the surface shall be an equilibrium surface under a rotation ω is found to be given by the system of equations

$$\frac{b_n}{n} = a_n\left(1 - \frac{\omega^2}{2\pi\rho}\right), \qquad (n = 1, 2, 3, \ldots).$$

The constancy of area of the curve (ii) can be effected by keeping c_1 constant. This method is subject to certain modifications, owing to the possibility of the various series becoming divergent.

The linear series of circles and ellipses (corresponding to the Maclaurin spheroids and Jacobian ellipsoids) are investigated without difficulty, and the points of bifurcation on these series are found. The first point of bifurcation on the latter series is shown to lead to a pear-shaped curve, similar to that of Poincaré, and it is shown that an exchange of stabilities takes place at this point.

The linear series of which this pear-shaped figure is the starting point can now be investigated, the equation being expanded in an ascending series of powers of a parameter θ. Since the equations are not linear, the calculation of terms multiplying high powers of θ is extremely laborious. The series is, therefore, calculated only as far as θ^5, this being found to give tolerable accuracy so far along the series as the expansion is required.

After passing through various pear-shaped configurations the fluid is found to assume a shape similar to that of a soda-water bottle with a somewhat rounded end. Beyond this the configuration is found to be suggestive of a tennis-racquet with a very short handle. A "neck" gradually forms at the point at which the handle joins the racquet, and this becomes more pronounced, until ultimately the curve separates into two parts.

As we proceed along this series the rotation steadily increases. At the point of bifurcation the value of $\omega^2/2\pi\rho$ is 0·375; when separation takes place this value is about 0·43. It is tolerably clear (although not rigorously proved) that when separation takes place, the primary may be regarded as the Jacobian ellipse, corresponding to rotation

$$\omega^2/2\pi\rho = 0 \cdot 43 \ldots \ldots \ldots \text{(iv)},$$

distorted by the tidal influence of the satellite. The linear diameters of primary and satellite are in a ratio of about 4 : 1.

The points of bifurcation on the Poincaré series are not investigated. Since the Jacobian ellipse determined by equation (iv) is known to be stable, there is ground for supposing that the series remains stable up to the point of separation. It therefore appears probable that the primary moves through a cycle of configurations in which Jacobi's and Poincaré's figures alternate. The angular momentum is decreased by about 30 per cent., at the ejection of each satellite.

"On the Action of the Spurge (*Euphorbia hiberna*, L.) on Salmonoid Fishes."* By H. M. KYLE, M.A., D.Sc., St. Andrews University. Communicated by Professor McINTOSH, F.R.S. Received June 25,—Read December 12, 1901.

Introduction.

It has been known for some years that the Irish peasantry employed a simple method of procuring salmon and trout through the agency of the Spurge (*E. hiberna*, L.). The plant cut into small pieces and pounded with stones, or simply trampled upon at some convenient spot on a river, forms an emulsion in the water which, being swept downward into the pools, carries death to all fishes in its course. The fatality thus produced seems to have been enormous—80 to 100 salmon are reported to have been killed at one time,† and again in the Bandon rivers 500 to 1000 fish of various descriptions are said to have been poisoned during one season.‡ In the light of the experiments to be recorded presently, these statements do not seem exaggerated, for the Spurge-extract, even in small quantities, is almost as fatal to fishes as corrosive sublimate.

The fatal effect of the Spurge on fishes has been known in other countries besides Ireland, but to what ingredient or ingredients of the plant these effects are due seems never to have been investigated. The following pages contain a brief record of experiments which, though incomplete in many ways, throw considerable light upon the action of the Spurge, and open out to view some interesting problems.

As the range of this research has included within its scope several

* The Fishmongers' Company generously gave a sum for the carrying out of this research. Special thanks are also due to the Hon. G. W. Hely Hutchison, secretary to the Irish Inland Fisheries Commission, who forwarded plants of Spurge from Ireland.

† 'Report of the Inspectors of Irish Fisheries,' 1898, p. 193.

‡ *Ibid.*, 1892, p. 53.